Faculty of Physics, Lomonosov Moscow State University

Advanced Quantum Field Theory: Modern Applications in HEP, Astro & Cond-Mat
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Handout 2 (Spring 2016 term)

1. Within the theory of a free massless scalar $D = 1 + 1$, consider the operator
$$:\hat{T}_{00}:(f) \equiv \frac{1}{2}\int :(\partial_t\hat{\varphi}(x,0))^2 + (\partial_x\hat{\varphi}(x,0))^2:\, f(x)\mathrm{d}x,$$
where $f(x)$ is a non-negative infinitely differentiable function with a compact support (a *bump function*). Find the norm of the state $:\hat{T}_{00}:(f)\,|0\rangle$ and the expectation value of $\hat{A}$ in the state
$$|\Phi_\alpha\rangle \equiv \mathcal{N}(1 + \alpha :\hat{T}_{00}:(f))\,|0\rangle\,, \qquad \alpha \in \mathbb{C},$$
where $\mathcal{N}$ is the normalization constant, in terms of the Fourier image of f. Using explicit instances of $f(x)$, show that the density expectation value in $|\Phi_\alpha\rangle$
$$\lim_{\mathrm{supp} f \to \{x\}} \quad \frac{\langle\Phi_\alpha|\ :\hat{T}_{00}:(f)\ |\Phi_\alpha\rangle}{\int f(x)\mathrm{d}x}.$$
is not positive-definite and is unbounded below.

2. Within the problem of an accelerated mirror in a $D = 1+1$ massless scalar theory, start with the integral expression for the regularized stress-energy tensor $\langle T_{\mu\nu}(x_-)\rangle$ given in lecture 7 and check the formulae for the renormalized stress-energy
$$\langle T_{00}^{\mathrm{ren}}\rangle = -\langle T_{01}^{\mathrm{ren}}\rangle = \frac{1}{12\pi}\sqrt{p'(x_-)}\left[\frac{1}{\sqrt{p'(x_-)}}\right]''.$$

3. Show that, when the effect of the external magnetic field on the photon dispersion is neglected, the photon splitting to more than 2 photons is kinematically forbidden, namely, the corresponding phase volumes vanish. Moreover, find the double-splitting momentum integral
$$V_{\mathrm{ph}}^{(2)}(\boldsymbol{k}) = \int d^3k_1 d^3k_2 \delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2 - \boldsymbol{k})\delta(|\boldsymbol{k}_1| + |\boldsymbol{k}_2| - |\boldsymbol{k}|).$$

4. Using the well-known expression for the Heisenberg–Euler effective action, find the term in the Lagrangian that is responsible for the photon splitting in a strong external magnetic field $\bar{B}$ (more precisely, for the contribution of the hexagon diagram to the amplitude).

5. Within a scalar $D = 1 + 1$ theory with the Lagrangian
$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi)^2 - \frac{\lambda}{4}(\varphi^2 - v^2)^2, \qquad m \equiv v\sqrt{\lambda},$$
find the mass counterterm $\delta\mathcal{L} = -\frac{1}{2}\delta m^2\varphi^2$ which renormalizes the self-energy, within the leading order in λ. In the integrals, use the momentum cutoff $|p| < \Lambda \to \infty$ and explicitly expand the counterterm in divergent powers of Λ.

6. Prove the Fierz identities ($\varphi, \chi, \psi, \omega$ are classical spinor fields)
$$\bar{\varphi}\gamma^\mu_{\mathrm{L}}\chi\ \bar{\psi}\gamma_{\mu\mathrm{L}}\omega = \bar{\varphi}\gamma^\mu_{\mathrm{L}}\omega\ \bar{\psi}\gamma_{\mu\mathrm{L}}\chi, \qquad \bar{\varphi}\gamma^\mu_{\mathrm{R}}\chi\ \bar{\psi}\gamma_{\mu\mathrm{L}}\omega = -2\bar{\varphi}P_{\mathrm{L}}\omega\ \bar{\psi}P_{\mathrm{L}}\chi,$$
where $P_{\mathrm{L,R}} \equiv (1 \mp \gamma_5)/2$, $\gamma^\mu_{\mathrm{L,R}} \equiv \gamma^\mu P_{\mathrm{L,R}}$.

7. Starting with the general expression for the 1-loop effective potential for axions in the AWZ model (see lecture 11), use the momentum cutoff regularization $|p| < \Lambda \to \infty$ and find the divergent and finite parts of the potential.