Faculty of Physics, Lomonosov Moscow State University

Advanced Quantum Field Theory: Modern Applications in HEP, Astro & Cond-Mat

Instructor: assist. prof., Dr. **Oleg Kharlanov**

Term 2 Examination Syllabus (Fall 2016)

1. Tight-binding approximation and its origins. A particle in a periodic potential well, crystal momentum, hopping integrals, lattice wave function. Bloch functions and the Brillouin zone. Second-quantized version of the TB model; linear closed array of atoms, the effective mass.

2. Self-consistent field approximation: the Hartree–Fock and Hartree–Fock–Roothaan equations; a qualitative analysis of the approximations used.

3. Fundamentals of the Density functional theory: the energy functional, the Hohenberg–Kohn theorems, the universal functional $F[n(\mathbf{x})]$. Density functionals and correlation functions. The Thomas–Fermi model.

4. A deeper look at the Density functional theory: the energy functional within the Kohn–Sham approach, the exchange-correlation potential and its approximations, the kinetic energy and the Kohn–Sham equations.

5. Lattice field theory: ‘free fields’. Second quantization of the TB model. one-particle eigenstates, lattice ‘field’ operator, the Fermi sea as a QFT vacuum, quasiparticles and anti-particles (holes). The 2-point Green’s function on a lattice.

6. Lattice field theory: symmetries. Symmetries of the TB model for a general periodic lattice and a bipartite lattice. Global $U(1)$ and $SU(2)$ symmetries; local spin; $U(1)$ charge & current density operators; the Peierls substitution and the local $U(1)$ gauge symmetry of the TB model.

7. Lattice field theory: ‘interacting fields’. The Coulomb interaction within the TB model and the role of selected Coulomb matrix elements. The Hubbard model, its symmetries and its weak-coupling regime, (on a half-filled 2D square lattice).

8. The strong-coupling regime of the Hubbard model at half-filling: the Heisenberg model.
The t–J model (qualitatively).

9. Fermi surface instabilities: susceptibilities and nesting vectors, an example of the Hubbard model on a 2D square lattice at half-filling. Mean-field theory for such a model around the ferromagnetic state and the corresponding critical on-site coupling.

10. Fermi surface instabilities: an example of an antiferromagnetic (Neél) order in the Hubbard model on a 2D square lattice at half-filling. The mean-field theory, the ‘effective Dirac equation’, and the gap equation. Estimation of the gap via the van Hove singularities of the DOS.

11. Bardeen–Cooper–Schrieffer theory of superconductivity: the short-range electron attraction in the second-quantized language, the expectation values (condensates) describing Cooper pairing, mean-field Hamiltonian, Bogolyubov transformation and quasiparticle spectrum, gap equation at zero and finite temperatures, critical temperature.

12. Graphene: the TB model for π-band electrons, 1-particle eigenstates; the Brillouin zone and the Fermi points; energy dispersion around the Fermi points.

13. Effective Dirac equation for the excitations about the Fermi points in graphene: the equation, the field operator, the charge and current density, the canonical commutation relations.

14. Graphene and (slowly-varying) external electromagnetic fields. The interacting theory for graphene and its Lagrangian formulation; time rescaling and the effective electromagnetic coupling.